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AMS 341 Final Cheat Sheet

Primal	**Dual**
$\max z = c^T x$	$\min w = b^T y$
$Ax \leq b$	$A^T y \geq c$
$x \geq 0$	$y \geq 0$

Symmetric Form Max problem, all constraints $\leq$, all vars ≥ 0

Weak Duality If x is feas to primal, y feas to dual, then $cx \leq by$ (obj of max $\leq$ obj of min)

Lemma If both equal, we're optimal

Cor If p unbounded, then dual is infeasible. (Not iff, both can be infeasible)

Strong Duality If P has finite optimum solution, dual also has finite optimal solution with equal value

Transportation Problem Supply must equal demand. Add dummys to correct if wrong.

	s1	s2	s3	Supply
Dem 1	Costs			
Dem 2				
Demand				

NW Corner Take NW Corner. Look at minimum of row or column. Eliminate which is minimum from future consideration. Populate cell with eliminated cost. Subtract that from noneliminated cost.

Min Cost Find whatever has minimum cost. Find min row or col and eliminate, adjusting costs accordingly.

Vogel Penalty Method For each row and col, penalty is difference between cheapest and second cheapest cost. Pick row or col with *largest* penalty. Pick cell of minimum cost in there.

Transportation Simplex

1. Find starting BFS.
2. Is BFS optimal? Stop if yes. O/w find entering variable
3. Find loop created by entering var
4. Δ = min of variable in loop that is being decreased
5. Apply delta appropriately to all cells
6. Remote one variable that becomes 0.

Note: BTPs are always feasible, never unbounded

Dual Sol if Primal Max

Opt val of d.v. y_i if const. $i \leq$=coef of s_i in opt row 0

if const. $i \geq$ const = -1*(coef of e_i in opt row 0)

if const i is eq const = (coef of a_i in opt row 0)-M

Dual Sol if Primal Min

Opt val of d.v. x_i if const $\leq$ = coef of s_i in opt row 0

if const i $\geq$ = -(coef of e_i in opt row 0

if const i = (coef of a_i in row 0) + M

Assignment Problem

Must be a square matrix - each cost is some rating, and we are allowed only a value of one. If we need more than one, we must create a duplicate

Hungarian Method

1. For each row, subtract smallest element from all.
2. Then, do same for columns.
3. Can we find 0s in each row, col. Find fewest possible lines.
4. If # lines < m find smallest uncovered element. Subtract it from all uncovered, add k to double covered.

Like BTP, this is always feasible, bounded, solution integer

Critical Path

Build a project network

$ET(1) = 0$; $ET(i) = max(ET(j) + \text{time } j \rightarrow i)$

$LT(Last) = ET(Last)$; $LT(i) = min(LT(j) - \text{time } i \rightarrow j$

$TF(i \rightarrow j) = LT(j) - ET(i) - task(i,j)$

Critical path has elements where TF=0. To design an LP, define a var for every node, min $x_n - x_1$.

Constraints are $x_j \geq x_i + c(i,j)$

Crashing the project: Design problem, adjust constraints with vars and add costs to obj.

Integer Programming

Variables allowed to be binary, use M in constraint for binary variables if you don't specify the word binary.

Branch and Bound Solve LP, then on non-integer values, create a computation tree and resolve

Cutting Plane Pick row in LP relaxation that has RHS remainder closest to .5. Rewrite as eqn with sum of vars as signed = RHS. Rewrite all LHS and RHS coef as some integer + some positive frac. Move all frac to RHS, all wholes to LHS. New RHS ≤ 0 is new constraint.

Dynamic Programming Ex: $f_i(s)$ = max grade in class i..3 given s TAs; Start at greatest value for i, ignoring stages other than that. Continue for stage $i - 1$. If we're doing a min problem, use min of all possibilities, otherwise, do max. Stop at desired solution for final stage - only have to calculate 1.