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Final Cheat Sheet

Solutions for $a_n = ca_{n/k} + f(n)$

c	$f(n)$	a_n
$c = 1$	d	$d \lceil \log_k n \rceil + A$
$c = k$	d	$An - \frac{d}{k-1}$
$c \neq k$	dn	$An^{\log_k c} + \left(\frac{kd}{k-c}\right)n$
$c = k$	dn	$dn(\lceil \log_k n \rceil + A)$

Solving Recurrence Relations

1. Check base cases for initial conditions.
2. Show that $a_{n+1} =$ yours with $n+1$

AND

$$N(\overline{A_1} \cap \overline{A_2} \cap \dots \overline{A_n}) = N - s_1 + s_2 - s_3 + \dots (-1)^n s_n$$

$$s_1 = N(A_1) + N(A_2) + \dots + N(A_n)$$

$$s_2 = N(A_1 \cap A_2) + \dots + N(A_{n-1} \cap A_n)$$

OR

$$N(A_1 \cup A_2 \cup \dots A_n) = s_1 - s_2 + s_3 + \dots (-1)^{n-1} s_n$$

Generating Functions

For odd or even problems, consider x^2 options

$$g(x) = (1 + x + x^2)(1 + x^2 + x^4)^3$$

Look for desired quantity r coef of x^r

$$\frac{1 - x^{n+1}}{1 - x} = 1 + x^2 + x^3 + \dots x^n \quad (1)$$

$$\frac{1}{1 - x} = 1 + x + x^2 + \dots \quad (2)$$

$$(1 + x)^n = 1 + \binom{n}{1}x + \binom{n}{2}x^2 + \dots \binom{n}{r}x^r + x^n \quad (3)$$

$$(1 - x^m)^n = 1 - \binom{n}{1}x^m + \binom{n}{2}x^{2m} \quad (4)$$

$$\frac{1}{(1 - x)^n} = 1 + \binom{1+n-1}{1}x + \binom{2+n-1}{2}x^2 + \dots + \binom{r+n-1}{r}x^r \quad (5)$$