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Midterm 2 Cheat Sheet

Tree Connected graph with no circuits

***m*-ary Tree** Each internal node has exactly m children

For m-ary, $n = mi + 1$

$n = i + \ell$

$h \geq \lceil \log_m \ell \rceil$

Height # of edges in longest path

Balanced Tree Tree balanced if all leaves at h or $h - 1$

For balanced m-ary, $h = \lceil \log_m \ell \rceil$

Minimum Spanning Trees

Kruskal: Sort by cost, add an edge if it will not create a circuit for all edges in order

Prim: start at any node. Add cheapest edge between a node in connected set and unconnected set

Quick TSP

Find an MST. Double it. Now you have an Euler cycle. Find a circuit. Delete repeated edges.

$quick\ cost \leq 2 * opt\ cost$

Addition Principle: If sets are disjoint, number of ways to select a single item is their sum

Multiplication principle: If composite outcomes distinct, total outcomes are product

Combination: ${}_nC_r = \frac{n!}{(n-r)!r!}$

Permutation: ${}_nP_r = \frac{n!}{(n-r)!}$

Set Composition: In using multiplication principle, we must be able to tell which part came from which step.

Arrange r_1 objects of type 1, $\ldots r_m$ objects of type m; Order not important; repeats allowed:
$P(n; r_1, r_2, \ldots r_m) = \frac{n!}{r_1!r_2!\ldots r_m!}$

Selection Form: r obj, n types
$\binom{r+n-1}{r}$

Repeats, Order Not Impt:
$\binom{r+n-1}{r}$

No repeats, order not impt:
$\binom{n}{r}$

Repeats, Order Important:
$P(n, r)$

No repeats, order impt:
n^r

Distibute r distinct obj, n boxes
n^r

Distribute r indistinct obj, n boxes
$\binom{r+n-1}{r}$

Equivalent forms:

Set 1: $\binom{r+n-1}{r}$

ways to select r obj from n types, rpts allowed, order not impt

ways to distribute r identical objects into n distinct boxes

sol to $\sum_{i=1}^{n} x_i = r$ where $x_i \geq 0, x_i \in \mathbb{Z}$

Set 2: n^r

Distribute r distinct obj into n distinct boxes

Select r items out of n types, rpts allowed, order impt

Make sequence of length r using n letters