AMS 301: Midterm 1 Review

Greg Cordts

July 5, 2010

$$G + \bar{G} = G_n$$

Circuit has no repeated nodes. Cycle has repeated nodes, not repeated edges.

$$2e = \sum deg(v)$$

Cor: The number of nodes of odd degree must be even.

Bipartite

Odd circuit ⇒ not bipartite
Bipartite ⇒ no odd circuit
A graph is bipartite iff all circuits have even length

Planar Graphs

Use circle chord method.

$$r = e - v + 2$$

$$e \leq 3v - 6$$

For planar, bipartite: $e \leq 2v - 4$

Latter Part

Euler Cycle cycle that visits each edge exactly once and each vertex at least once.
Euler cycle ⇔ 1. every node has even degree 2. graph connected

Euler Trail Euler cycle that does NOT return to start node. Euler trail but NOT euler cycle: Nec and suff ⇔ 1. exactly 2 nodes of odd degree 2. connected

Hamilton Circuit Visits each NODE exactly once

Nec: Connected, all nodes degree ≥ 2

Suff: (Dirac) If G is graph on at least 3 nodes ($n \geq 3$) and all nodes have $deg \geq \frac{n}{2}$ then G has HC.

Rule 1: If vertex x has degree 2, both edges incident must be part of any circuit.

Rule 2: No proper subcircuit can be formed.

Rule 3: Once a node is used, discard remove from consideration all edges not used.

Thm 0: If the amount of k nodes removed results in $> k$ connected components, G does not have a hamilton circuit.

Thm 5: If a bipartite grpah G has $|L| \neq |R|$, no HC.

Vizing Theorem Max node degree $\leq$ edge coloring number $\leq$ max node degree + 1

Four-Color Theorem Every planar graph can be 4 node colored by at most 4 nodes

Art gallery theorem Triangulate room, think of as a graph where sides are edges and nodes are corners. Node color the graph. Color used least often is minimum number of guards.