# Final Review Sheet

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## 1 Review Material

### 1.1 Integration By Parts

Select a $f, dg$. Then $fg - \int g * df$

### 1.2 Trig Formulas

$\sin(2A) = 2 \sin A \cos A$

### 1.3 Directional Derivative

If $\vec{u}$ is a unit vector:

$$f_{\vec{u}}(a,b) = f_x(a,b)u_1 + f_y(a,b)u_2$$

### 1.4 Extreme Value Theorem

$f$ has a global max, min in region R iff:

- f(x,y) continuous
- R is closed
- R is bounded

## 2 Non-Cartesian Coordinates

### 2.1 Polar Coordinates

$$x = r \cos\theta$$

$$y = r \sin\theta$$

$$x^2 + y^2 = r^2$$

$$dA = r\ drd\theta = rd\theta dr$$

### 2.2 Cylindrical Coordinates

$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$z = z$$

$$x^2 + y^2 = r^2$$

$$dV = r dr d\theta dz$$

### 2.3 Spherical Coordinates

$$x = \rho\sin\phi\cos\theta$$

$$y = \rho\sin\phi\sin\theta$$

$$z = \rho\cos\phi$$

$$x^2 + y^2 + z^2 = \rho^2$$

$$dV = \rho^2\sin\phi d\rho d\phi d\theta$$

## 3 New Material

### 3.1 Flow Lines

A flow line is a path $\vec{r}(t)$ whose velocity vector equals $\vec{v}$.

$$\vec{r}'(t) = \vec{v} = \vec{F}(\vec{r}(t))$$

### 3.2 Parameterization

#### 3.2.1 Parameterization of a Line

$$x = x_0 + at, y = y_0 + bt, z = z_0 + ct$$

when parallel to vector $a\vec{i} + b\vec{j} + c\vec{k}$

#### 3.2.2 Length of a Curve

$$Length(C) = \int_a^b \|\vec{v}\|\, dt$$

### 3.3 Vector Fields

#### 3.3.1 Definition

A vector field in $n$-space is a function $\vec{F}(x, \ldots)$ whose value at a point is an $n$-dimensional vector.

## 3.4 Line Integrals

### 3.4.1 Formula

Second p

$$\int_C \vec{F} \cdot d\vec{r}$$

For a smooth parameterization:

$$\int_C \vec{F} \cdot d\vec{r} = \int_a^b \vec{F}(\vec{r}(t)) \cdot \vec{v}(t)\, dt$$

### 3.4.2 Significance

**Force Field** Line integral is total work

**Oriented Closed curve** Line integral is circulation

### 3.4.3 Fundamental Theorem of Calculus

If $C$ is piecewise smooth and orinted starting at P, ending at Q, and if f is a function whose gradient is continuous on C:

$$\int_C \nabla f \cdot d\vec{r} = f(Q) - f(P)$$

## 3.5 Path Independence

Path indpt if any piecewise, smooth path between two points has the same value.

Continous $\vec{F}$ is path indpt iff:

- $\vec{F}$ is a gradient field.

Vector field path indpt iff $\int_C \vec{F} \cdot d\vec{r} = 0$ for every closed curve C.
If $\vec{F} = \nabla f$, f is the **potential function** of $\vec{F}$.

## 3.6 Curl (Green's Theorem)

If $\vec{F}$ is a gradient field with continous partial derivatives, and if C is closed, its scalar curl is 0.

$$\int_C \vec{F} \cdot d\vec{r} = \int_R \underbrace{\left(\frac{\delta F_2}{\delta x} - \frac{\delta F_1}{\delta y}\right)}_{\text{scalar curl}} dxdy$$

## 3.7 Flux Integral

$$\vec{A} = A\vec{n}$$
$$\int_S \vec{F} \cdot d\vec{A}$$

#### 3.7.1 For a surface $z = f(x,y)$

$$\int_R \vec{F}(x,y,f(x,y)) \cdot (-f_x\vec{i} - f_y\vec{j} + \vec{k})\, dxdy$$

#### 3.7.2 For a cylinder, oriented away from z-axis

$$\int_{z_i}^{z_f} \int_{\theta_i}^{\theta_f} \vec{F}(R,\theta,z) \cdot (\cos\theta\vec{i} + \sin\theta\vec{j}) R\, d\theta dz$$

#### 3.7.3 For a Parameterization

$$\int_R \vec{F}(\vec{r}(s,t)) \cdot \left(\frac{\delta\vec{r}}{\delta s} \times \frac{\delta\vec{r}}{\delta t}\right) dsdt$$

Surface area:

$$\int_R \left\|\frac{\delta\vec{r}}{\delta s} \times \frac{\delta\vec{r}}{\delta t}\right\| dsdt$$

### 3.8 Divergence

$$\text{div}\vec{F} = \nabla \cdot \vec{F} = \frac{\delta F_1}{\delta x} + \frac{\delta F_2}{\delta y} + \frac{\delta F_3}{\delta z}$$

#### 3.8.1 Divergence Theorem

If W is solid, has boundry S outward (closed, piecewise, smooth), and F smooth vector field on open region contianing W, S.

$$\int_S \vec{F} \cdot d\vec{A} = \int_W \text{div}\vec{F}\, dV$$