AMS 261: Midterm 2 Review

Greg Cordts

December 19, 2009

1 Derivatives in Higher Degrees

1.1 Chain Rule

$$\frac{dz}{dt} = \frac{\delta z}{\delta x}\frac{dx}{dt} + \frac{\delta z}{\delta y}\frac{dy}{dt}$$

1.2 Taylor Approximations

$$f(x,y) \approx L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

$$f(x,y) \approx Q(x,y) = L(x,y) + \frac{f_{xx}(a,b)}{2}(x-a)^2 + f_{xy}(a,b)(x-a)(y-b) + \frac{f_{yy}(a,b)}{2}(y-b)^2$$

1.3 Differentiability

Arrows ONLY go in the following directions:

$$\text{f:differentiable} \Rightarrow \text{f:continuous}$$
$$\text{f:differentiable} \Rightarrow f_x \text{ and } f_y \text{ exist}$$
$$\text{f:differentiable} \Leftarrow \begin{cases} f_x, f_y \text{exist} \\ f_x, f_y \text{continuous} \end{cases}$$

2 Optimization

2.1 Extreme Value Theorem

f must have its global max, min in region R iff:

- $f(x,y)$ is continuous
- R is closed
- R is bounded

2.2 Local Extrema

- Find critical points of f
- Compute D and f_{xx} at critical points
- Use second derivative test

2.3 Second Derivative Test

D	f_{xx}	Conclusion
$D > 0$	$f_{xx} > 0$	local min
$D > 0$	$f_{xx} < 0$	local max
$D < 0$		saddle point
$D = 0$		inconclusive

2.4 Constrained Optimization

$$\nabla f = \lambda \nabla g$$

λ is the rate of change of the optimal value of f as c increases

3 Double Integrals

3.1 Definition

$$\int_R f dA = lim_{\Delta x, \Delta y \to 0} \sum_{1 \leq i \leq n, 1 \leq, j \leq m} f(u_{ij}, v_{ij}) \Delta x \Delta y$$

3.2 Non-Rectangular Regions

- Inner integral must have variable
- Outer integral must be a number

3.3 Polar Coordinates

$$x = r \cos\theta$$

$$y = r \sin\theta$$

$$x^2 + y^2 = r^2$$

$$dA = r\ drd\theta = rd\theta dr$$

3.4 Cylindrical Coordinates

$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$z = z$$
$$x^2 + y^2 = r^2$$
$$dV = rdrd\theta dz$$

3.5 Spherical Coordinates

$$x = \rho\sin\phi\cos\theta$$
$$y = \rho\sin\phi\sin\theta$$
$$z = \rho\cos\phi$$
$$x^2 + y^2 + z^2 = \rho^2$$
$$dV = \rho^2\sin\phi d\rho d\phi d\theta$$