Final Study Sheet - AMS 210

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1 Eigenvalues

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

For eigenvalues, do $A - \lambda\mathbb{I}$.

$$A - \lambda\mathbb{I} = \begin{bmatrix} a - \lambda & b \\ c & d - \lambda \end{bmatrix}$$

Find determinant, then solve for λs.

2 Linear Regression

$$x' = x - \bar{x}$$

Device to remember: None.

3 Absorbing State

Introduce absorbing state into FINAL value.

3.1 IROQ

Remap as

$$\begin{matrix} \mathbb{I} & \mathbb{R} \\ \emptyset & \mathbb{Q} \end{matrix}$$

matrix!

Device to remember: I Rock!

3.2 Find N matrix

$$N = (\mathbb{I} - \mathbb{Q})^{-1}$$

Device to remember: **N**ine **E**lephants **I**nvented **I**ced **M**agic **Q**uilts (N equals Inverse of Identity minus Q)

3.3 Number of Rounds to Switch

$$\vec{1}\mathbb{N} = [a\ b]$$

3.4 Expected time in any particular round

Look at $\mathbb{N}$.

4 Leslie Model

4.1 Growth Multiplier

Find largest eigenvalue! Then turn into a percent!

4.2 Long Term Distribution

Find eigenvector for that eigenvalue. Then, make it so that $|\vec{x}|_s = 1$.

5 Basis of Vector Space

5.1 Basis of Range

Pivot until you can't. The linearly independent COLUMNS are your vector space. Remember, the dimension of the vector space is the rank.

5.2 Basis of the Null Space

Take your upper triangular matrix and set it equal to zero. Solve in terms of the linearly dependent variable. The coefficients are the basis.

5.3 Another Vector in Range

$$x * (\in range) = x'(\in range) + x^0(\in null)$$

Remember, you can multiply $x^0$by anything and it's still in the null space.

6 Pseudoinverse

$$B^+ = (B^T B)^{-1} B^T$$

Remember: AP=IATA AT

6.1 Cosine Formula and Correlation Coefficient

$$cor(\vec{a}, \vec{b}) = \frac{\vec{a} \cdot \vec{b}}{|a|_e |b|_e}$$

$$\cos\theta = \frac{a \cdot b}{|a|_e |b|_e}$$

7 QR Decomposition

To find each column of q, $q_n^C = a_n^C$minus a's projection on all prior q.

NOTE: In manipulating q, you can reduce - relationships between are important!

Then, normalize it:

$$q_n^* = \frac{q_n}{|q_n|_e}$$

Remember, euclidean norm:

$$|a|_e = \sqrt{a \cdot a}$$

So

$$Q = \left[\begin{array}{cc} q_1^* & | q_2^* \end{array} \right]$$

Since Q is orthonormal,

$$Q^T = Q^{-1}$$

Remember, $A = QR$. Use that fact to find $A^{-1}$and R. ($A^{-1} = R^{-1} Q^T$).

8 Legendre Polynomials

For each term:

$$w_n = \frac{\int_a^b f(x) L_n dx}{\int_a^b L_n L_n dx}$$

Then, the polynomial is the linear combination of:

$$\sum_n w_n L_n$$